

Nonadiabatic effects in the dynamics of collapsing Bose-Einstein condensates

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The imploding dynamics of ⁸⁵Rb condensates is obtained in terms of the nonadiabatic dynamical characteristics of the homoclinic bifurcation giving rise to the collapse of the cloud. The agreement with the recent experimental observation of the imploding dynamics is excellent.

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The collapse of trapped Bose-Einstein condensates (BEC) with attractive interactions is a complex phenomenon that has generated a special interest [1–4]. It occurs when the attractive interactions overwhelm the kinetic zero-point energy of the atoms. A BEC can only avoid implosion when the following condition is satisfied:

$$\frac{N_0|a|}{kd} < 1, \quad (1)$$

where a is the s -wave scattering length, N_0 is the number of atoms in the condensate, $d = \sqrt{\hbar/m\omega}$ is the harmonic oscillator length (ω being the geometric mean of the trap frequencies), and k is the dimensionless stability coefficient which depends on the geometry of the magnetic trap [5]. BEC with attractive interactions have been observed for the first time in ⁷Li for which the scattering length is negative ($a = -1.45$ nm) [2]. In these experiments, the initial number of condensate atoms is close to the critical value ($N_c \cong 1200$) determined by Eq. (1) and the collapse events are triggered by thermal fluctuations or by macroscopic quantum tunneling [6–8]. As a result of the coupling with the large number of noncondensate particles, the system exhibits a complex dynamical behavior characterized by the collapse and the growth cycles [9,10]. Recently, near a Feshbach resonance new experiments have explored the implosion dynamics induced by switching the s -wave scattering length of ⁸⁵Rb atoms a from positive to a final value $a_{collapse}$ less than that of the threshold value $a_c = -kd/N_0$ [11]. This technique which produces large nearly pure condensates ($T=0; N_0 > N_c$) allows well controlled experimental studies of the onset of the instability. These results also provide a stringent test for the mean-field theory [12] based on the Gross-Pitaevskii equation with loss processes which can be written in the following form:

$$i\hbar \frac{\partial \Psi}{\partial t} = \left[-\frac{\hbar^2}{2m} \nabla^2 - U|\Psi|^2 + \frac{m\omega r^2}{2} \right] \Psi - i\hbar [K_2|\Psi|^2 + K_3|\Psi|^4] \frac{\Psi}{2}, \quad (2)$$

where $U = 4\pi\hbar^2|a|/m$, K_2 and K_3 denote the two-body dipolar and the three-body recombination loss-rate coefficients respectively.

Once the scattering length is shifted to a value more negative than a_c , the cloud undergoes a compression controlled by the attractive interactions. It is well established that in these systems the inelastic collisions are unimportant in the early stages of the implosion; they only act later to limit the growth of the central peak [13,14]. As a result these recombination processes do not affect the nature of the instability giving rise to the collapse which can be captured by applying a time dependent variational approach [15]. The evolution equation for the radius of a spherical cloud is therefore [8,14]

$$\frac{\partial^2 R}{\partial \tau^2} = -R + \frac{1}{R^3} - \frac{2\gamma}{R^4}, \quad (3)$$

where $\gamma = N_0|a|/\sqrt{2\pi}d$ is a measure of the magnitude of the attractive interactions; the length and time scales are normalized in units of d and ω^{-1} , respectively. This equation admits two equilibrium points $R_+ > R_-$, where R_+ is the radius of the metastable condensate that exists when $a > a_c$ and R_- corresponds to the unstable solution which plays the role of a boundary between collapsing and noncollapsing behaviors. When the intensity of the attractive interactions is increased, the two solutions come closer together and merge at $a = a_c$. After a sudden jump of the scattering length to $a_{collapse} < a_c$, the relaxation to the remnant condensate is first dominated by the slow transit of the trajectory in the vicinity of the instability point. When the amplitude of the quench is sufficiently small, i.e., $(a_{collapse} - a_c)/a_c \ll 1$, the

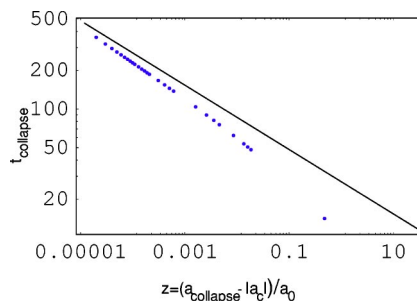


FIG. 1. (Color online) Evolution of the incubation period $t_{collapse}$ as a function of $a_{collapse}$ close to the HSN point for $N_0 = 6000$. The periods computed from Eq. (5) and from numerical simulations of Eq. (3) are given by the solid line and the circles, respectively.

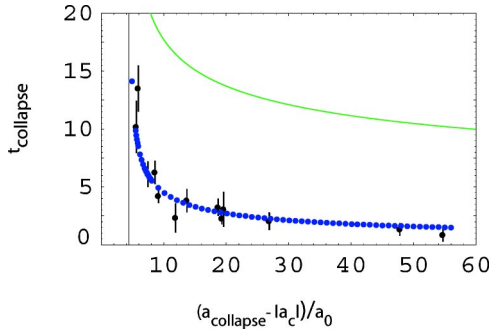


FIG. 2. (Color online) Evolution of the incubation period $t_{collapse}$ as a function of $a_{collapse}$ close to the HSN point for $N_0 = 6000$. The periods computed from Eq. (5) and Eq. (3) are given by the solid line and the circles, respectively. The vertical line indicates the critical value a_c . The dots with error bars are the experimental observation points (Fig. 2 of Ref. [11])

dynamics is governed by the following Hamiltonian saddle-node (HSN) normal form. Taking advantage of the slow time scale of the “order parameter” q , the fast evolving modes may be adiabatically eliminated (multiple-scale methods [16,17]) to obtain

$$\frac{\partial^2 q}{\partial \tau^2} = 10 \left[\gamma - \gamma_c + \frac{q^2}{R_c} \right], \quad (4)$$

where $q = R_c - R$, $R_c = (5)^{-1/4}$ and $\gamma_c = 2/5R_c$. Near this limit point, the system presents a slow power law decrease proportional to $(\gamma - \gamma_c)t^2$. Such plateau behavior that lasts for some time after the jump of the value of a corresponds to an incubation period $t_{collapse}$ that dramatically increases when the scattering length $a_{collapse}$ approaches the critical value a_c . An integration of Eq. (4) yields the following expression for the plateau lifetime

$$t_{collapse} = \int_0^\infty \frac{1}{\sqrt{20 \left(\frac{q^3}{3R_c} - q(\gamma_c - \gamma) \right)}} dq = \frac{C}{\omega} \left(\frac{d}{N_0 a_0} \right)^{1/4} \left[\frac{a_0}{|a_{collapse}| - |a_c|} \right]^{1/4}, \quad (5)$$

where $C = 3^{1/4} R_c^{1/4} [\Gamma(1/4)\Gamma(5/4)] / (5\pi)^{1/2}$ and a_0 is the Bohr radius.

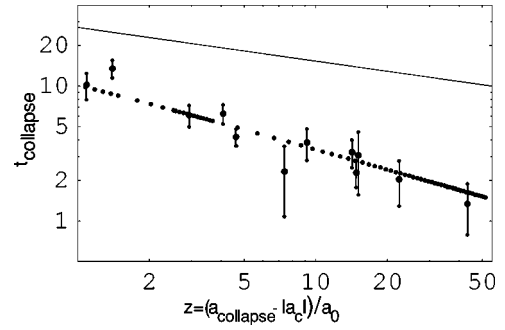


FIG. 3. (Color online) Same as Fig. 2 in log-log plot.

In analogy with the theory of the critical phenomena [18], the dependence of $t_{collapse}$ on $a_{collapse}$ can be characterized by a universal critical index n ,

$$t_{collapse} \sim [|a_{collapse}| - |a_c|]^n, \quad (6)$$

where $n = -1/4$. This scaling law is displayed in logarithmic scale in Fig. 1 and is in agreement with the collapse time obtained from numerical simulations of Eq. (3) when $|a_{collapse}|$ is very close to $|a_c|$. Nevertheless, this region has not been investigated in the experiments. The experimental data recently obtained by Donley *et al.* are located for large jump values of the scattering length [11]. In this region, the normal form, given by Eq. (4) is not sufficient to describe the dynamical behavior of the system. One must revert to Eq. (3) that controls the complete dynamical behavior of the HSN bifurcation. This equation contains all higher-order corrections including the so-called nonadiabatic corrections (in the sense that they do not result from the multiple scale method used in moving from Eq. (3) to Eq. (4) [19]). In this region, numerical simulations of Eq. (3) show that we switch from a $n = -1/4$ scaling law to a regime with $n \approx -1/2$ ($n = -0.4936$ fitting on the numerical results). The results, represented in Figs. 2 and 3 for the log-log scaling law are in excellent agreement with the experimental data.

Therefore, the nonadiabatic effects correct the scaling law in such a way that far from the HSN point the dynamics behaves as around a saddle-node bifurcation point in a dissipative system [20–22].

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